Formal Language

$\Sigma = \{a, b\}$

come from alphabet denoted as Σ

String: sequence of $\emptyset$ or more symbols (from an alphabet Σ)

the string of $\emptyset$ symbol is denoted ε

string on $\Sigma = \{a, b\}$:

ε, a, b, aa, bb, ab, abbb, ...

Σ^* ≡ set of all strings on Σ

$\Sigma = \{a, b\} = \{0, 1\} = \{a, \varepsilon\} = \{A\text{-}Z, a\text{-}z, 0\text{-}9, ..\}$

A language (on Σ) is a set of strings (on Σ)

Operation on strings

Suppose α is a string: abab

β is a string: aaab

$\alpha\beta$ is the concatenation: ababaaab

$\alpha^2 = \alpha\alpha$ = abababab → associative (order doesn't matter)

Let L be a language

L_1 ''

L_2 ''

$L_1 L_2$ is the concatenation of L_1 and L_2

$= \{\alpha\beta \mid \alpha \in L_1, \beta \in L_2\}$

$L_1 = \{\varepsilon, a, b\}$

$L_2 = \{bc, aaab\}$

$L_1 L_2 = \{bc, aaab, abc, bbc, baaab, aaaab\}$

$L_1 L_1 = \{\varepsilon, a, b, aa, ab, ba, bb\}$

$\alpha\varepsilon = \varepsilon\alpha = \alpha$ (ε acts as 1)

$L_3 = \{a, b\}$

L_3^k = all length $-k$ string on $\Sigma = \{a, b\}$

$L_3^3 = \{aaa, aab, aba, baa, abb, bab, bba, bbb\}$

$L^1 = L$

$L^0 = \{\varepsilon\} \rightarrow L^0 M = M L^0 = M$

$\emptyset m = m\emptyset = \emptyset$

Regular Expression on Σ

each regular expression denotes a language Σ

reg. expr	language
ε	$L_\varepsilon = \{\varepsilon\}$
α	$L_\alpha = \{\alpha\}$
$\gamma\delta$	$L_\gamma L_\delta$
γ^k	L_γ^k
$\gamma \mid \delta$	$L_\gamma \cup L_\delta = \{a\} \cup \{b\}$
γ^*	$L_\gamma^* = \{L^0 \cup L^1 \cup L^2 \cup L^3 \cup \ldots\}$