

Regular Languages

order of
precedence

Reg. expr	language
ϵ	$\{\epsilon\}$
a	$\{a\}$
④ $\alpha\beta$	$L_\alpha L_\beta$
② α^*	L_α^*
③ αB	$L_\alpha \cup L_B$
① (α)	L_α

α^k

$$L_\alpha^k = L_\alpha^{k-1} L_\alpha$$

α^0

$$\{\epsilon\} = L_\alpha^0$$

$\alpha^? / [\alpha]$

$$\{\epsilon\} \cup L_\alpha$$

optional

one or more

$\alpha^+ = \alpha\alpha^*$

$$L_\alpha L_\alpha^*$$

positive closure

$$\alpha \bowtie \beta = (\alpha\beta\alpha)^*$$

$$[a \dots x] = a | b | c | \dots | x$$

\uparrow character closure \uparrow or of all characters

$$[N\sigma \dots x] = c_1 | c_2 | c_3 | \dots | c_k$$

where $\{c_1, \dots, c_k\} = \Sigma$ all characters in $[\]$

$L_1 \cup L_2$ is a regular language : $re_1 | re_2$ ($\forall L_1, L_2$ regular)

$L_1 \cap L_2$ is a regular language

$$\tau_1 = \Sigma^* | L_1$$

Formalism for regular sets:

① regular expression

② DFA - Deterministic Finite Automata

↳ this machine has states and transitions

③ NFA - Nondeterministic Finite Automata

④ Left-linear Grammar

↳ $M \rightarrow a$

$M \rightarrow aN$; where N is a non-terminal

A DFA $M = (Q, \Sigma, \emptyset, s, F)$

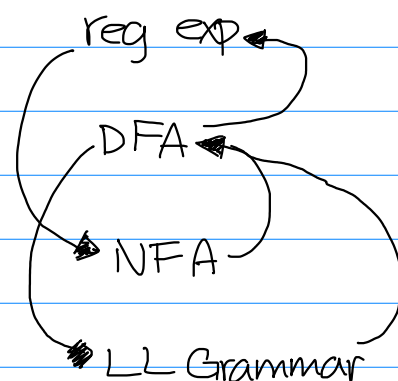
$Q = \{q_0, q_1, \dots, q_k\}$ set of states

Σ is an alphabet

$\emptyset = Q \times \Sigma \rightarrow Q$ transition function

$s \in Q$ start of the state

$F \in Q$ final state(s)



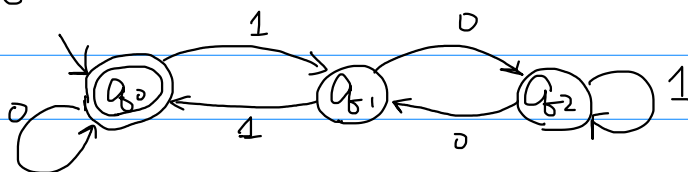
Example: Suppose $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{0, 1\}$

\emptyset	0	1
q_0	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

$s = q_0$

$F = \{q_0\}$

Diagram: state \bigcirc , start state \hookrightarrow , final state \bigcirc



* this machine should accept any binary # of multiple of 3

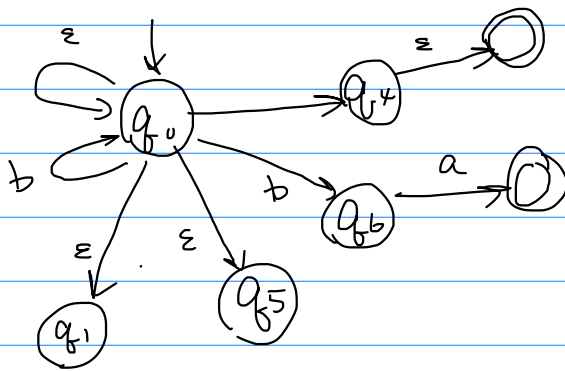
reg. set = $\{11, 1001, 110, \dots\}$

$q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_1$

rejected because q_1 is not a final state!!

NFA $\rightarrow \emptyset : Q \times (|\epsilon| \cup \Sigma) \rightarrow 2^2 \leftarrow$ This only change from DFA

	ϵ	a	b	...
q_0	$\{q_0, q_1, q_5\}$	$\{q_3\}$	$\{q_0, q_4, q_6\}$	
q_1				
i				
q_k				



$\{q_0, q_1, q_5\}$